De-Neutrosophication of Linear Heptagonal and Nonagonal Neutrosophic Numbers using the Removal Area Method: A Comprehensive Analysis and Numerical Validation

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Abstract

The existence of three disjoint portions makes neutrosophic set theory a significant topic today, clarifying muddled information and providing academics with a wide range of applications in several fields. In broad terms, neutrosophic sets are an expanded form of crisp sets, fuzzy sets, and intuitionistic fuzzy sets that focus on the reluctant, vague, and unclear aspects of real-world mathematical problems. This study paper centers explicitly on the illustration of heptagonal and nonagonal neutrosophic numbers and their categorization in various aspects. An implementation of the linear heptagonal and nonagonal neutrosophic numbers de-neutrosophication approach, utilizing the removal area method, has been devised here, and it significantly impacts the crispness of the heptagonal and nonagonal numbers.

Keywords: Neutrosophic set, Heptagonal neutrosophic number, Nonagonal neutrosophic numbers, Deneutrosophication

1. Introduction

Quantifying uncertainty is a key part of current scientific research, and it leads to new mathematical frameworks for dealing with indeterminacy all the time. Zadeh's ground breaking 1965 invention of fuzzy sets [1] started this change by making partial truth values between 0 and 1 officials. This made it possible to deal with imprecise events in a strict way. This base led to a series of generalizations: Atanassov's intuitionistic fuzzy sets [2] included clear degrees of non-membership; Chen used these ideas for reliability analysis [3]; and Yen et al. created triangular fuzzy regression models [5]. By the end of the 1990s, trapezoidal [6] and pentagonal [4] representations had come out. These representations allowed for more detailed modeling of uncertainty in fields ranging from supply chain optimization [7] to medical diagnostics. Smarandache's neutrosophic framework [8–9] brought about a paradigm shift by adding three independent dimensions' truth (T), indeterminacy (I), and falsity (F) to model complex real-world uncertainties where elements may show partial membership, non-membership, and indeterminacy at the same time. This three-part structure was much better at expressing ideas than previous models [10], especially when there was contradictory evidence [14, 19], when transitioning IoT systems [20, 23], when choosing medical devices [19], and when assessing sustainability [30]. Soon after, simplified [12] and single-valued [16] versions came out. Chakraborty et al. set up formal operations for triangular/trapezoidal neutrosophic numbers [16,18,21]. Recent additions, such as bipolar representations [21] and pentagonal structures [18, 24], have shown better performance in transportation logistics [24] and economic modeling [25].

Recent discoveries show how important advanced neutrosophic structures are becoming. Ahmed and Khan's 2023 geometric decomposition method [27] showed that higher-order defuzzification was more accurate, but it also showed that nonagonal systems had problems with computing. At the same time, Chen and Wang [28] showed that nonagonal neutrosophic sets are very good at environmental impact assessments because they can capture intricate interconnections of ecological uncertainty that typical models ignore. The 2024 healthcare research by Gupta and Sharma [29] confirmed that heptagonal fuzzy numbers are better for making resource allocation choices that need to reflect uncertainty in a nuanced way. These improvements show how important it is to have specialist de-neutrosophication algorithms that work with higher-order structures.

Even with these changes, complicated situations like multi-stage illness progression [15], supply chain disruptions that change a lot, and precise sustainability benchmarking [30] show that current representations still have problems. Heptagonal (7-point) and nonagonal (9-point) neutrosophic numbers fill in this gap by allowing for full parameterization of changing states of uncertainty, high-resolution vagueness resolution, and complex representation of overlapping uncertainty domains things that simpler forms can't do. These higher-order structures are especially good at simulating nonlinear uncertainty progression in medical diagnostics, multi-tier supply networks, and climatic systems where trapezoidal representations don't work [11]. But the main problem with de-neutrosophication the process of turning complicated neutrosophic structures into clear, actionable values has made it hard for people to use them in real life. For asymmetric distributions, well-known methods like Center-of-Area (COA) don't work [16], Basic Defuzzification Distribution (BADD) becomes too expensive to compute [21], and the Expected Value Method (EQM) loses important modal information [18]. For heptagonal and nonagonal structures, these constraints become worse in a way that makes it very hard to use them in the actual world, as shown by recent computer research [27].

This study fills up that gap with a geometrically based de-neutrosophication framework made just for higher-order neutrosophic numbers. Our removal area method is new because it divides heptagonal and nonagonal structures into trapezoidal segments based on optimized inflection points ($\alpha = 1/3, \beta = 2/3$ for heptagonal; $\alpha = 1/4, \beta = 1/2, \gamma = 3/4$ for nonagonal). Then, it averages the left and right areas under the T, I, and F membership functions. The closed-form solutions that come out of this are Equation (9), which combines 21 heptagonal parameters, and Equation (10), which combines 27 nonagonal variables. These transformations are computationally efficient and have been tested by many numerical tests. Comprehensive testing shows three important benefits: consistency (clear values match the central tendencies of the input parameters), scalability (the ability to handle 27-dimensional spaces efficiently), and monotonicity (outputs reflect input magnitudes in a proportional way, such as 12.57 vs. 6.37 for high/low parameter sets). These features make possible things that were not possible before, such as modelling 7-stage disease progression with uncertainty bands [29], measuring disruption risks across 9-tier supply networks, and showing how different environmental uncertainty domains affect climate modelling [28].

This study makes a big step forward for uncertainty-aware systems by connecting the theoretical expressiveness of higher-order neutrosophic numbers with their real-world computing uses. The suggested "removal area method" provides a strong structure for de-neutrosophication, making it possible to turn complicated heptagonal and nonagonal neutrosophic numbers into clear, usable values. This breakthrough has big effects on how people make decisions in the real world, such as when they plan for ethical AI development, sustainable infrastructure, and other areas where uncertainty is crucial.
This discovery not only moves the area of neutrosophic mathematics forward, but it also presents engineers, economists, and climate scientist's important new tools to address uncertainties that are becoming more and more complicated. The capacity to turn complicated neutrosophic structures into understandable numbers makes them more useful for multi-criteria decision-making, optimization issues, and predictive modeling.
Additionally, the results of this study are significant beyond just neutrosophic theory. The de-neutrosophication processes will be very advantageous for mathematicians, researchers, and practitioners who work with heptagonal and nonagonal numbers. The methods and ideas provided here may also lead to new uses in fields like geometry, number theory, and scientific modeling, which will encourage people from other fields to work together and push the boundaries of uncertainty quantification.
This work paves the way for further exploration of more intricate neutrosophic structures and their potential applications in real-world scenarios. It also strengthens the idea that neutrosophic mathematics is an important part of current uncertainty analysis.

Prior research

The development of uncertainty theory is very important for several reasons, including its role in creating useful scientific and mathematical models, modeling engineering structures, and solving multi-criteria issues. Recently, there has been talk about how useful it is to classify a heptagonal and nonagonal neutrosophic number in any area of study. How can we logically and scientifically turn a heptagonal or nonagonal neutrosophic number into its exact equivalent? How can we create a de-neutrosophication procedure that will engage people's interest?
The development of uncertainty theory is crucial for creating effective scientific and mathematical models, designing engineering structures, and addressing problems that involve multiple criteria. In recent talks, a lot of focus has been on figuring out how to classify heptagonal and nonagonal neutrosophic numbers in different areas of study. The most important question is how we may logically and scientifically turn a heptagonal or nonagonal neutrosophic number into its clear equivalent. Furthermore, a motivated de-neutrosophication procedure has to be put in place to make sure that the changed numbers still have meaning and use.

To use heptagonal and nonagonal neutrosophic numbers in real life, you need to be able to answer these questions. Translating these numbers into their exact counterparts makes them easier to understand and helps them fit into current mathematical systems. We may say that there is a reason to use heptagonal and nonagonal neutrosophic numbers and that they work with regular arithmetic operations by creating a systematic de-neutrosophication procedure. The most important thing about this discovery is that it might help people comprehend and use heptagonal and nonagonal neutrosophic numbers in a wider range of fields. We can deal with the problems that come up while using these figures by coming up with a logical and scientifically sound way to make them crispy and a compelling way to deneutrosophicate them. This study has real-world effects on how to predict uncertainties, make smart choices, and solve difficult issues in various fields.

2.1 Requirement for heptagonal and Nonagonal number

The heptagonal and nonagonal fuzzy numbers make it more likely that they reflect wrong information, which leads to the creation of inventive solutions to a variety of real-world problems. But when you compare it to the original triangular or trapezoidal fuzzy number, it's evident that formation has its own set of problems and issues with definition. Heptagonal and nonagonal show all the important information, and the hazy portion may be portrayed in a way that is both true and realistic. This approach of calculating and manipulating will be more useful to the decision-maker than any other fuzzy representation. It may be applied in a number of different fields, such as optimization approaches and addressing problems in economics, where we need additional parameters. If there are more than one variable, utilizing triangular or trapezoidal fuzzy numbers to show the assignment difficulty might be a problem. Therefore, heptagonal or nonagonal fuzzy numbers may assist in addressing the problem.

Using heptagonal and nonagonal fuzzy numbers makes it more likely that you will properly express ambiguous information, which leads to innovative solutions to many real-world problems. However, it is harder to define and create heptagonal and nonagonal fuzzy numbers than it is to define and create the original triangular or trapezoidal fuzzy numbers. Heptagonal and nonagonal fuzzy numbers provide all the important information, which makes it possible to show ambiguity in a way that is both accurate and realistic. This way of calculating and manipulating fuzzy data will be more useful to decision-makers than other fuzzy representations. It may be applied in a lot of different fields, such as optimization methods and problem-solving in economics, where more parameters are needed. In certain cases, when there are more variables, utilizing triangular or trapezoidal fuzzy numbers to show the assignment difficulty will cause a problem. Because of this, heptagonal or nonagonal fuzzy numbers may help solve these kinds of problems.

2.2. Limitations of Fuzzy logic

To deal with the problems that fuzzy logic has with neutral or indeterminate states, Smarandache [8] came up with the word "neutrosophic," which comes from the word "neutrosophy," which means "knowledge of neutral thought." The main difference between fuzzy sets and neutrosophic sets is that neutrosophic sets have a third part that is clearly defined: the neutral/indeterminate/unknown dimension. Fuzzy sets only use a membership function (truth), whereas intuitionistic fuzzy sets include a non-membership function (falsehood). Neutrosophic sets, on the other hand, combine three separate functions: truth (T), indeterminacy (I), and falsity (F). This three-part structure makes it possible to describe uncertainty in a more detailed way by capturing levels of membership, non-membership, and indeterminacy all at once. So, neutrosophic sets get beyond the limitations of standard fuzzy systems when it comes to expressiveness, especially when there is contradicting or partial information [9, 21]. Their ability to reflect complicated real-world uncertainties where things might show partial truth, partial untruth, and indeterminacy at the same time gives them a clear edge over traditional fuzzy and intuitionistic fuzzy sets.

3. Mathematical preliminaries

Definition 3.1. Neutrosophic set

A set $\widetilde{\mathrm{M_{Neu}}}$ in the universal discourse $\breve{\mathrm{X}}$, which is indicated by the generic symbol $\breve{\mathrm{x}}$ is said to be a neutrosophic set [9], if $\widetilde{\mathrm{M_{Neu}}} = \{\breve{\mathrm{x}}; [\rho_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}), \sigma_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}), \tau_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}})]: \breve{\mathrm{x}} \in \breve{\mathrm{X}}\}$, where $\rho_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}): \breve{\mathrm{X}} \to [0,1]$ is referred to as the truth membership function, which illustrates the level of certainty, $\sigma_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}): \breve{\mathrm{X}} \to [0,1]$ is referred to as the indeterminacy membership function, and $\tau_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}): \breve{\mathrm{X}} \to [0,1]$ is known as the falseness membership function, and it illustrates how unsure a judgment the decision maker is making is.

$\rho_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}), \sigma_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}})$ and $\tau_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}})$ Satisfies the following relation

$0 \le \rho_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}), \sigma_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}), \tau_{\widetilde{\mathrm{M_{Neu}}}}(\breve{\mathrm{x}}) \le 3+$

Definition 3.2. Single-valued Neutrosophic set

Neutrosophic set $\widetilde{M_{Neu}}$ in the previous definition 2.1 is called a Single Valued Neutrosophic Set $\widetilde{M_{Neu}}$ if x is a single valued independent variable. Thus

$\widetilde{M_{Neu}} = \{\breve{x}; [\rho_{\widetilde{M_{Neu}}}(\breve{x}), \sigma_{\widetilde{M_{Neu}}}(\breve{x}), \tau_{\widetilde{M_{Neu}}}(\breve{x})]: \breve{x} \in \breve{X}\}$

Where $\rho_{\widetilde{M_{Neu}}}(\breve{x}), \sigma_{\widetilde{M_{Neu}}}(\breve{x})$ and $\tau_{\widetilde{M_{Neu}}}(\breve{x})$ represents, respectively, the truth, indeterminacy, and falsity membership function. as stated in the previous definition, and the connection between the truth, indeterminacy, and falsity membership function is

$0 \le \rho_{\widetilde{M_{Neu}}}(\breve{x}), \sigma_{\widetilde{M_{Neu}}}(\breve{x}), \tau_{\widetilde{M_{Neu}}}(\breve{x}) \le 3.$

Suppose there are three points r_0, s_0, t_0 for which $\rho_{\widetilde{M_{Neu}}}(r_0) = 1, \sigma_{\widetilde{M_{Neu}}}(s_0) = 1, \tau_{\widetilde{M_{Neu}}}(t_0) = 1$ then M_{Neu} is called neut – normal.

A M_{Neu} is said to be neut-convex if it satisfies the following criteria, which implies that it is a subset of a real line:

1. $\rho_{\widetilde{M_{Neu}}} < \varphi s_1 + (1-\varphi)s_2 > \ge \min < \rho_{\widetilde{M_{Neu}}}(s_1), \rho_{\widetilde{M_{Neu}}}(s_2) >$
1. $\sigma_{\widetilde{M_{Neu}}} < \varphi s_1 + (1-\varphi)s_2 > \ge \min < \sigma_{\widetilde{M_{Neu}}}(s_1), \sigma_{\widetilde{M_{Neu}}}(s_2) >$
2. $\tau_{\widetilde{M_{Neu}}} < \varphi s_1 + (1-\varphi)s_2 > \ge \min < \tau_{\widetilde{M_{Neu}}}(s_1), \tau_{\widetilde{M_{Neu}}}(s_2) >$

   Where $s_1 \& s_2 \in R$ and $\varphi \in [0,1]$.

Definition 3.3. Single-valued Heptagonal neutrosophic number

A single-valued neutrosophic Heptagonal neutrosophic number is defined as

$\breve{H} = \langle [(\breve{i}^1, \breve{j}^1, \breve{k}^1, \breve{l}^1, \breve{m}^1, \breve{n}^1, \breve{o}^1); \mu], [(\breve{i}^2, \breve{j}^2, \breve{k}^2, \breve{l}^2, \breve{m}^2, \breve{n}^2, \breve{o}^2); \pi], [(\breve{i}^3, \breve{j}^3, \breve{k}^3, \breve{l}^3, \breve{m}^3, \breve{n}^3, \breve{o}^3); \rho]$

where $\mu, \pi, \rho \in [0,1]$

where $\mu, \pi, \rho \in [0,1]$ truth membership function $\mu_{\breve{H}}: R \to [0, a]$, the indeterminacy membership function $\pi_{\breve{H}}: R \to [b, 1]$, the falseness membership function

$\rho_{\breve{H}}(\breve{x}): R \to [c, 1]$ is given as

$$\mu_{\breve{H}}(\breve{x}) = \begin{cases} \mu_{\widetilde{Hs_1}}(x), \breve{i}^1 \le \breve{x} \le \breve{j}^1 \\ \mu_{\widetilde{Hs_2}}(x), \breve{j}^1 \le \breve{x} \le \breve{k}^1 \\ \mu_{\widetilde{Hs_3}}(x), \breve{k}^1 \le \breve{x} \le \breve{l}^1 \\ a, \breve{x} = \breve{l}^1 \\ \mu_{\widetilde{Ht_3}}(x), \breve{l}^1 \le \breve{x} \le \breve{m}^1 \\ \mu_{\widetilde{Ht_2}}(x), \breve{m}^1 \le \breve{x} \le \breve{n}^1 \\ \mu_{\widetilde{Ht_1}}(x), \breve{n}^1 \le \breve{x} \le \breve{o}^1 \\ 0, \text{otherwise} \end{cases}$$

$$\pi_{\breve{H}}(\breve{x}) = \begin{cases} \pi_{\widetilde{Hs_1}}(\breve{x}), \breve{i}^2 \le \breve{x} \le \breve{j}^2 \\ \pi_{\widetilde{Hs_2}}(\breve{x}), \breve{j}^2 \le \breve{x} \le \breve{k}^2 \\ \pi_{\widetilde{Hs_3}}(x), \breve{k}^2 \le \breve{x} \le \breve{l}^2 \\ b, \breve{x} = \breve{l}^2 \\ \pi_{\widetilde{Ht_3}}(\breve{x}), \breve{l}^2 \le \breve{x} \le \breve{m}^2 \\ \pi_{\widetilde{Ht_2}}(\breve{x}), \breve{m}^2 \le \breve{x} \le \breve{n}^2 \\ \pi_{\widetilde{Ht_1}}(\breve{x}), \breve{n}^2 \le \breve{x} \le \breve{o}^2 \\ 1, \text{otherwise} \end{cases}$$

$$\rho_{\breve{H}}(x) = \begin{cases} \rho_{\widetilde{Hs_1}}(\breve{x}), \breve{i}^3 \le \breve{x} \le \breve{j}^3 \\ \rho_{\widetilde{Hs_2}}(\breve{x}), \breve{j}^3 \le \breve{x} \le \breve{k}^3 \\ \rho_{\widetilde{Hs_3}}(\breve{x}), \breve{k}^3 \le \breve{x} \le \breve{l}^3 \\ c, \breve{x} = \breve{l}^3 \\ \rho_{\widetilde{Ht_3}}(\breve{x}), \breve{l}^3 \le \breve{x} \le \breve{m}^3 \\ \rho_{\widetilde{Ht_2}}(\breve{x}), \breve{m}^3 \le \breve{x} \le \breve{n}^3 \\ \rho_{\widetilde{Ht_1}}(\breve{x}), \breve{n}^3 \le \breve{x} \le \breve{o}^3 \\ 1, \text{otherwise} \end{cases}$$

Definition 3.4. Single-valued Nonagonal neutrosophic number

A single-valued neutrosophic Nonagonal neutrosophic number is defined as

$\breve{N} = \langle [(\breve{i}^1, \breve{j}^1, \breve{k}^1, \breve{l}^1, \breve{m}^1, \breve{n}^1, \breve{o}^1, \breve{p}^1, \breve{q}^1); \sigma], [(\breve{i}^2, \breve{j}^2, \breve{k}^2, \breve{l}^2, \breve{m}^2, \breve{n}^2, \breve{o}^2, \breve{p}^2, \breve{q}^2)$

where $\sigma, \tau, \pi \in [0,1]$ truth membership function $\sigma_{\breve{N}}: R \to [0, e]$, the indeterminacy membership function $\tau_{\breve{N}}: R \to [f, 1]$, the falseness membership function

$\varphi_{\breve{N}}(\breve{x}): R \to [g, 1]$ is given as

$$\sigma_{\breve{N}}(\breve{x}) = \begin{cases} \sigma_{\widetilde{Ns_1}}(\breve{x}), \breve{i}^1 \le \breve{x} \le \breve{j}^1 \\ \sigma_{\widetilde{Ns_2}}(\breve{x}), \breve{j}^1 \le \breve{x} \le \breve{k}^1 \\ \sigma_{\widetilde{Ns_3}}(\breve{x}), \breve{k}^1 \le \breve{x} \le \breve{l}^1 \\ \sigma_{\widetilde{Ns_4}}(\breve{x}), \breve{l}^1 \le \breve{x} \le \breve{m}^1 \\ e, \breve{x} = \breve{m}^1 \\ \sigma_{\widetilde{Nt_4}}(\breve{x}), \breve{m}^1 \le \breve{x} \le \breve{n}^1 \\ \sigma_{\widetilde{Nt_3}}(\breve{x}), \breve{n}^1 \le \breve{x} \le \breve{o}^1 \\ \sigma_{\widetilde{Nt_2}}(\breve{x}), \breve{o}^1 \le \breve{x} \le \breve{p}^1 \\ \sigma_{\widetilde{Nt_1}}(\breve{x}), \breve{p}^1 \le \breve{x} \le \breve{q}^1 \\ 0, \text{otherwise} \end{cases}$$

$$\tau_{\widetilde{N}}(\breve{x}) = \begin{cases} \tau_{\widetilde{Ns_1}}(\breve{x}), \breve{i}^2 \le \breve{x} \le \breve{j}^2 \\ \tau_{\widetilde{Ns_2}}(\breve{x}), \breve{j}^2 \le \breve{x} \le \breve{k}^2 \\ \tau_{\widetilde{Ns_3}}(\breve{x}), \breve{k}^2 \le \breve{x} \le \breve{l}^2 \\ \tau_{\widetilde{Ns_4}}(\breve{x}), \breve{l}^2 \le \breve{x} \le \breve{m}^2 \\ \quad f, \breve{x} = \breve{m}^2 \\ \tau_{\widetilde{Nt_4}}(\breve{x}), \breve{m}^2 \le \breve{x} \le \breve{n}^1 \\ \tau_{\widetilde{Nt_3}}(\breve{x}), \breve{n}^2 \le \breve{x} \le \breve{o}^2 \\ \tau_{\widetilde{Nt_2}}(\breve{x}), \breve{o}^2 \le \breve{x} \le \breve{p}^2 \\ \tau_{\widetilde{Nt_1}}(\breve{x}), \breve{p}^2 \le \breve{x} \le \breve{q}^2 \\ \quad 1, \text{otherwise} \end{cases}$$

$$\varphi_{\widetilde{N}}(\breve{x}) = \begin{cases} \varphi_{\widetilde{Ns_1}}(\breve{x}), \breve{i}^3 \le \breve{x} \le \breve{j}^3 \\ \varphi_{\widetilde{Ns_2}}(\breve{x}), \breve{j}^3 \le \breve{x} \le \breve{k}^3 \\ \varphi_{\widetilde{Ns_3}}(\breve{x}), \breve{k}^3 \le \breve{x} \le \breve{l}^3 \\ \varphi_{\widetilde{Ns_4}}(\breve{x}), \breve{l}^3 \le \breve{x} \le \breve{m}^3 \\ \quad f, x = \breve{m}^3 \\ \varphi_{\widetilde{Nt_4}}(\breve{x}), \breve{m}^3 \le \breve{x} \le \breve{n}^3 \\ \varphi_{\widetilde{Nt_3}}(\breve{x}), \breve{n}^3 \le \breve{x} \le \breve{o}^3 \\ \varphi_{\widetilde{Nt_2}}(\breve{x}), \breve{o}^3 \le \breve{x} \le \breve{p}^3 \\ \varphi_{\widetilde{Nt_1}}(\breve{x}), \breve{p}^3 \le \breve{x} \le \breve{q}^3 \\ \quad 1, \text{otherwise} \end{cases}$$

4. Material and Methods: De-neutrosophication of linear Heptagonal number and Nonagonal neutrosophic number

Researchers struggled to determine the best way to convert the heptagonal and nonagonal neutrosophic numbers to crisp numbers in this neutrosophic arena. Three distinct membership functions are present in both heptagonal and nonagonal neutrosophic numbers. This study suggests the "removal area method" to convert neutrosophic numbers into crisp numbers.

According to the outcomes of the heptagonal and nonagonal neutrosophic numbers and their membership functions, researchers have been striving to determine the most effective approach for converting these fuzzy numbers into crisp numbers. Several practical methods have been formulated over time to address this challenge, including the BADD method, CDD method, COA method, COG method, BOA method, EQM method, FCM method, and more.

The conversion of heptagonal and nonagonal neutrosophic numbers to crisp numbers has drawn the attention of many neutrosophic researchers. Three different membership functions can be seen for heptagonal and nonagonal neutrosophic numbers. This research suggests the "removal area method" to transform neutrosophic numbers into crisp numbers. In order to produce explicit representations, this method attempts to separate and extract the pertinent information from the neutrosophic numbers. The removal area method can significantly overcome the difficulty of converting heptagonal and nonagonal neutrosophic numbers into crisp numbers in the neutrosophic arena. This method provides a systematic and helpful method for deriving sharp values from neutrosophic numbers, allowing for a more straightforward interpretation and application of the obtained results. Consider the following linear heptagonal neutrosophic number.

$$\widehat{\mathbf{H}}_{\mathbf{neutro}} = (r_1, r_2, r_3, r_4, r_5, r_6, r_7; s_1, s_2, s_3, s_4, s_5, s_6, s_7; t_1, t_2, t_3, t_4, t_5, t_6, t_7)$$

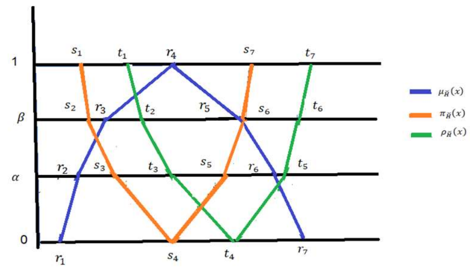


Figure 1. Graph illustration of the linear heptagonal neutrosophic number.

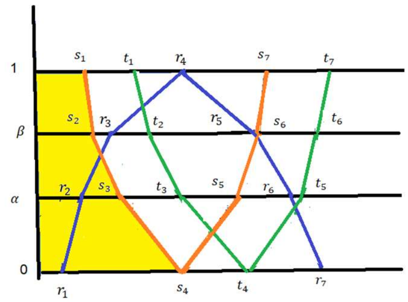
4.1. Description of the Graphical Representation and Area Calculation for Linear Heptagonal Neutrosophic Numbers

4.1.1. Graphical Representation of the Linear Heptagonal Neutrosophic Number

The linear heptagonal neutrosophic number, shown graphically, was the main subject of the above illustration. The truth membership function is represented by the heptagonal with blue lines. Heptagons with red and green lines illustrate the number's falsehood and indefiniteness membership functions, respectively. The main focus of the above illustration was to depict the graphical representation of the linear heptagonal neutrosophic number. The heptagon with blue lines visually represents the truth membership function. On the other hand, the heptagons with red and green lines illustrate the number's falsehood membership function and indefiniteness membership function, respectively.

4.1.2. Area Calculation Methodology for De-Neutrosophication

For the blue line defined heptagons, let's suppose a real number $w \in R$ and a fuzzy number $\breve{X}$. The fuzzy number's left side is represented by the region of dispersion on the left side of $\breve{X}$ in relation to w is $A_{\text{neurto l}}(\breve{X}, w)$ .Continuing in this manner,



the right zone area $\breve{X}$ in relation to w is , $A_{\text{neurto r}}(\breve{X}, w)$ For the leftmost top and inverted heptagon, taking into account a real number $w \in R$ as well as the fuzzy number $\breve{Y}$. The area of the least side of the $\breve{Y}$ in relation to w is defined as $A_{\text{neurto l}}(\breve{Y}, w)$,the area enclosed by and the left section of the fuzzy number when a real number is also taken into consideration for the leftmost top and inverted heptagons. For the rightmost top and inverted heptagons, the area of the right side of the $\breve{Y}$ in relation to w is again an ambiguous measurement, and the area bounded by the leftmost top and is defined as $A_{\text{neurto r}}(\breve{Y}, w)$. The area has also been established for $\breve{Z}$ with regard to w.

When referring to Average,

$$A_{\text{neutro}}(\breve{X}, w) = \frac{A_{\text{neutro l}}(\breve{X}, w) + A_{\text{neutro r}}(\breve{X}, w)}{2} \text{------(1)}$$

$$A_{\text{neutro}}(\breve{Y}, w) = \frac{A_{\text{neutro l}}(\breve{Y}, w) + A_{\text{neutro r}}(\breve{Y}, w)}{2} \text{---- (2)}$$

$$A_{\text{neutro}}(\breve{Z}, w) = \frac{A_{\text{neutro l}}(\breve{Z}, w) + A_{\text{neutro r}}(\breve{Z}, w)}{2} \text{-----(3)}$$

The value of the de-neutrosophication of a linear heptagonal neutrosophic number was then determined as

$$A_{\text{neutro}}(\widetilde{D_{\text{hep}}}, w) = \frac{A_{\text{neutro}}(\breve{X}, w) + A_{\text{neutro}}(\breve{Y}, w) + A_{\text{neutro}}(\breve{Z}, w)}{3} \text{------- (4)}$$

When w = 0 in equations (1), (2) ,(3) and (4) we get,

$$A_{\text{neutro}}(\breve{X}, 0) = \frac{A_{\text{neutro l}}(\breve{X}, 0) + A_{\text{neutro r}}(\breve{X}, 0)}{2} \text{----------------------- (5)}$$

$$A_{\text{neutro}}(\breve{Y}, 0) = \frac{A_{\text{neutro l}}(\breve{Y}, 0) + A_{\text{neutro r}}(\breve{Y}, 0)}{2} \text{--- (6)}$$

$$A_{\text{neutro}}(\breve{Z}, 0) = \frac{A_{\text{neutro l}}(\breve{Z}, 0) + A_{\text{neutro r}}(\breve{Z}, 0)}{2} \text{--- (7)}$$

$$A_{\text{neutro}}(\widetilde{D_{\text{hep}}}, 0) = \frac{A_{\text{neutro}}(\breve{X}, 0) + A_{\text{neutro}}(\breve{Y}, 0) + A_{\text{neutro}}(\breve{Z}, 0)}{3} \text{------------ (8)}$$

Let $\tilde{L} = (r_1, r_2, r_3, r_4, r_5, r_6, r_7)$, $\tilde{M} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ and $\tilde{N} = (t_1, t_2, t_3, t_4, t_5, t_6, t_7)$

Figure 2

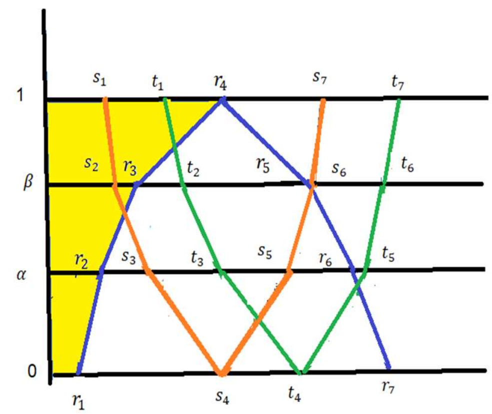


Figure 3

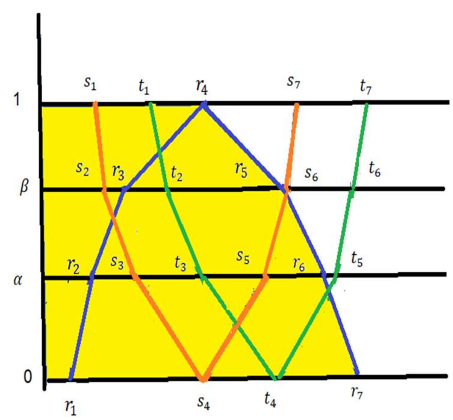
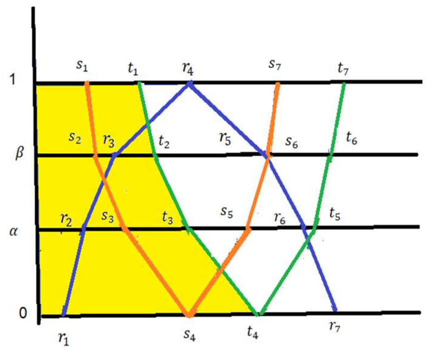


Figure 4

Figure 5



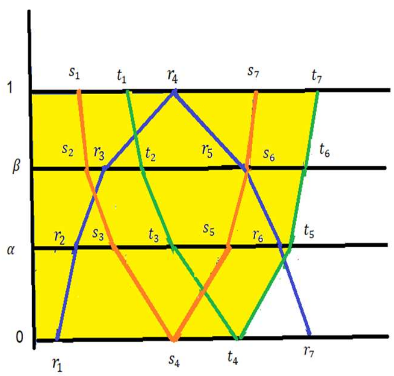
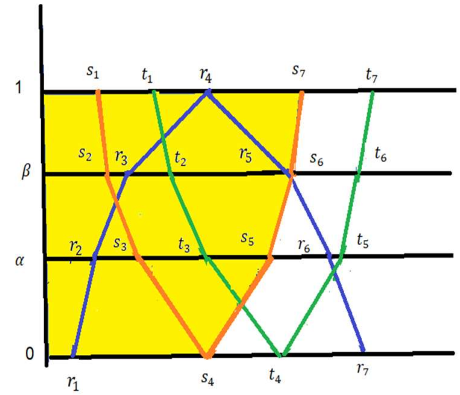


Figure 6

$$A_{neutro}(\breve{X},0) = \text{area of figure 2}$$
$$= \frac{(r_1+r_2)\alpha}{2} + \frac{(r_2+r_3)(\beta-\alpha)}{2} + \frac{(r_3+r_4)(1-\beta)}{2}$$

$$A_{neutro}(\breve{X},0) = \text{area of figure 3}$$
$$= \frac{(r_6+r_7)\alpha}{2} + \frac{(r_5+r_6)(\beta-\alpha)}{2} + \frac{(r_4+r_5)(1-\beta)}{2}$$

$$A_{neutro}(\breve{Y},0) = \text{area of figure 4}$$
$$= \frac{(s_1+s_2)(1-\beta)}{2} + \frac{(s_2+s_3)(\beta-\alpha)}{2} + \frac{(s_3+s_4)\alpha}{2}$$



$$A_{neutro}(\breve{Y},0) = \text{area of figure 5}$$
$$= \frac{(s_6+s_7)(1-\beta)}{2} + \frac{(s_5+s_6)(\beta-\alpha)}{2} + \frac{(s_4+s_5)\alpha}{2}$$

$$A_{neutro}(\check{Z},0) = \text{area of figure 6}$$
$$= \frac{(t_1+t_2)(1-\beta)}{2} + \frac{(t_2+t_3)(\beta-\alpha)}{2} + \frac{(t_3+t_4)\alpha}{2}$$

$$A_{neutro}(\check{Z},0) = \text{area of figure 7}$$
$$= \frac{(t_6+t_7)(1-\beta)}{2} + \frac{(t_5+t_6)(\beta-\alpha)}{2} + \frac{(t_4+t_5)\alpha}{2}$$

$$A_{neutro}(\breve{X},0)$$
$$= \frac{\frac{(r_1+r_2)\alpha}{2} + \frac{(r_2+r_3)(\beta-\alpha)}{2} + \frac{(r_3+r_4)(1-\beta)}{2} + \frac{(r_6+r_7)\alpha}{2} + \frac{(r_5+r_6)(\beta-\alpha)}{2} + \frac{(r_4+r_5)(1-\beta)}{2}}{2}$$

$$A_{neutro}(\breve{Y},0)$$
$$= \frac{\frac{(s_1+s_2)(1-\beta)}{2} + \frac{(s_2+s_3)(\beta-\alpha)}{2} + \frac{(s_3+s_4)\alpha}{2} + \frac{(s_6+s_7)(1-\beta)}{2} + \frac{(s_5+s_6)(\beta-\alpha)}{2} + \frac{(s_4+s_5)\alpha}{2}}{2}$$

$$A_{neutro}(\check{Z},0)$$
$$= \frac{\frac{(t_1+t_2)(1-\beta)}{2} + \frac{(t_5+t_6)(\beta-\alpha)}{2} + \frac{(t_4+t_5)\alpha}{2} + \frac{(t_6+t_7)(1-\beta)}{2} + \frac{(t_5+t_6)(\beta-\alpha)}{2} + \frac{(t_4+t_5)\alpha}{2}}{2}$$

$$A_{neutro}(D_{hep},0) = \frac{\begin{gathered}(r_1+r_2+r_6+r_7+s_3+2s_4+s_5+t_3+2t_4+t_5)\alpha+\\(r_2+r_3+r_5+r_6+s_2+s_3+s_5+s_6+t_2+t_3+t_5+t_6)(\beta-\alpha)+\\(2r_4+r_3+r_5+s_1+s_2+s_6+s_7+t_1+t_2+t_6+t_7)(1-\beta)\end{gathered}}{12}$$

Where $0<\alpha<\beta<1$

When $\alpha=\frac{1}{3},\beta=\frac{2}{3}$

$$A_{neutro}(D_{hep},0) = \frac{\left(\begin{gathered}r_1+r_7+s_1+s_7+t_1+t_7)+\\2(r_2+r_3+r_4+r_5+r_6+s_2+s_3+s_4+s_5+s_6+t_2+t_3+t_4+t_5+t_6+t_7)\end{gathered}\right)}{36} \quad (9)$$

4.1.3. Numerical Computation Data for Linear Heptagonal Neutrosophic Numbers

This section presents the direct numerical results obtained by applying the "removal area method" to several linear heptagonal neutrosophic numbers. The study material indicates that Table 1 is the primary source of this information. In this table, each row stands for a different linear heptagonal neutrosophic number. Its 21 parameters (seven for the truth membership function, seven for the indeterminacy, and seven for the falseness function) carefully characterize it. The table also displays the equivalent crisp de-neutrosophication value, as determined by Equation (9), a crucial finding.

Table 1. Numerical computation of linear Heptagonal neutrosophic number.

S. No	Linear Heptagonal neutrosophic number	De-neutrosophication value
1	(1,2,4,8,9.5,13,14.5; 3.5,5,7.5,9,10.5,11; 6,6.5,7,9.5,13.5,15,15.5)	8.2777
2	(0.5,1.5,3.5,7.5,9,12.5,14; 2.5,3,4.5,7,8.5,10,10.5; 5.5,6,6.5,9,13,14.5,15)	7.7777
3	(1.5,2.5,4.5,8.5,10,13.5,15; 3.5,4,5.5,8,9.5,11,11.5; 6.5,7,7.5,10,14,15.5,16)	8.7777
4	(0.7,1.7,3.7,7.7,9.2,12.7,14.2; 2.7,3.2,4.7,7.2,8.7,10.2,10.7; 5.7,6.2,6.7,9.2,13.2,14.7,15.2)	7.9777
5	(5,6,8,12,13.5,17,18.5; 7,7.5,9,11.5,13,14.5,15; 10,10.5,11,13.5,17.5,19,19.5)	12.2777

A look at the de-neutrosophication values in Table 1 shows that the crisp outputs always lie within a useful range compared to the input parameters of the heptagonal neutrosophic numbers. For example, the crisp values usually match up with the central tendency or the areas with the most input parameters throughout the truth, indeterminacy, and falsity functions. This consistency shows that the approach can efficiently turn many different kinds of uncertainty into one easy-to-understand number. These clear values are necessary for doing basic algebra, making straightforward comparisons, and helping with ranking in decision-making processes when the intrinsic complexity of neutrosophic numbers would make it impossible.

Table 1 shows a clear and continuous trend: the size of the de-neutrosophication value is closely related to the size of the input parameters that define the linear heptagonal neutrosophic number. For instance, when you look at S. No 2 and S. No 5, the input parameters for S. No 2 are typically lower (e.g., the truth function begins at 0.5, the indeterminacy at 2.5, and the falsehood at 5.5) than those for S. No 5 (e.g., the truth function starts at 5, the indeterminacy at 7, and the falsity at 10). The de-neutrosophication value for S.No 2 is 7.7777, while for S.No 5 it is 12.2777. This comparison makes it evident that the crisp de-neutrosophication value likewise goes up when the numerical values of the defining parameters for the heptagonal neutrosophic number grow. This trend is valid for all the instances in Table 1. This discovered association is a highly desirable and logical characteristic for any method of de-neutrosophication. It demonstrates that the approach effectively conveys the "magnitude" or "central tendency" of the uncertain number, transforming a vague concept of "larger" or "smaller" uncertainty into a precise value that corresponds to the degree of uncertainty. This dependability is essential for these de-neutrosophicated numbers to be useful in situations where comparisons and rankings need to be meaningful.

Consider a linear Nonagonal neutrosophic number

$\widehat{N}_{neutro} = (l_1,l_2,l_3,l_4,l_5,l_6,l_7,l_8,,l_9; m_1,m_2,m_3,m_4$

, $\mathrm{m}_5, \mathrm{m}_6, \mathrm{m}_7, \mathrm{m}_8, \mathrm{m}_9; \mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3, \mathrm{n}_4, \mathrm{n}_5, \mathrm{n}_6, \mathrm{n}_7.\mathrm{n}_8, \mathrm{n}_9$)

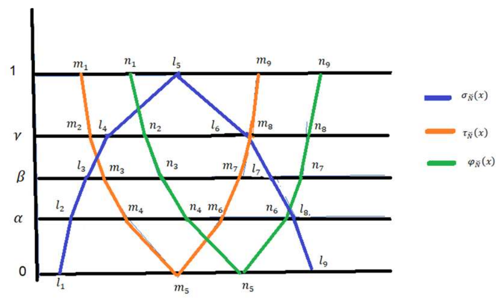


Figure 7. Graphical representation of linear nonagonal neutrosophic number.

4.2. Description of the Graphical Representation and Area Calculation for Linear Nonagonal Neutrosophic Numbers

4.2.1. Graphical Representation of the Linear nonagonal Neutrosophic Number

The linear nonagonal neutrosophic number graph was the focus of the picture above. The blue lines on the nonagon show the true membership function. Nonagons with red and green lines show the number's false and indefinite membership functions, respectively.

4.2.2. Area Calculation Methodology for De-Neutrosophication

Let $\tilde{\mathrm{L}} = (\mathrm{l}_1, \mathrm{l}_2, \mathrm{l}_3, \mathrm{l}_4, \mathrm{l}_5, \mathrm{l}_6, \mathrm{l}_7, \mathrm{l}_{8,,}\mathrm{l}_9)$,

$\widetilde{\mathrm{M}} = (\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_6, \mathrm{m}_7, \mathrm{m}_8, \mathrm{m}_9)$ and

$\widetilde{\mathrm{N}} = (\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3, \mathrm{n}_4, \mathrm{n}_5, \mathrm{n}_6, \mathrm{n}_7.\mathrm{n}_8, \mathrm{n}_9)$

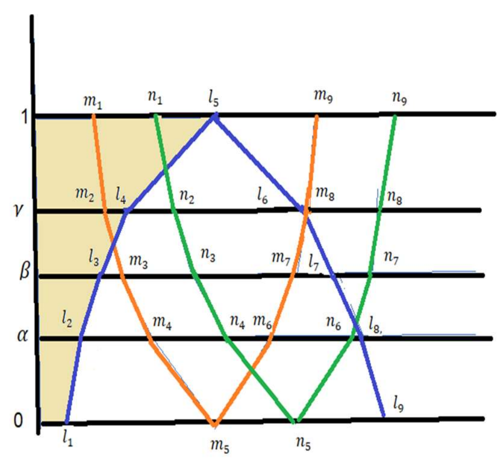


Figure 8

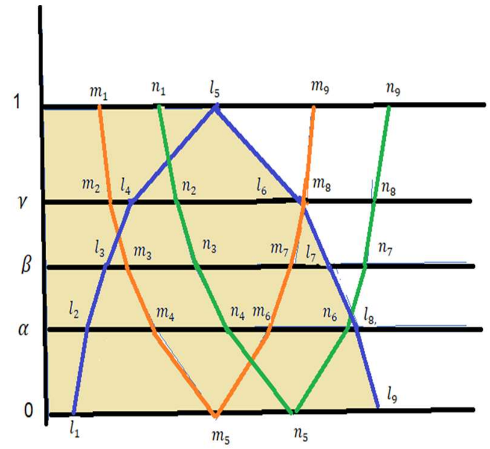


Figure 9

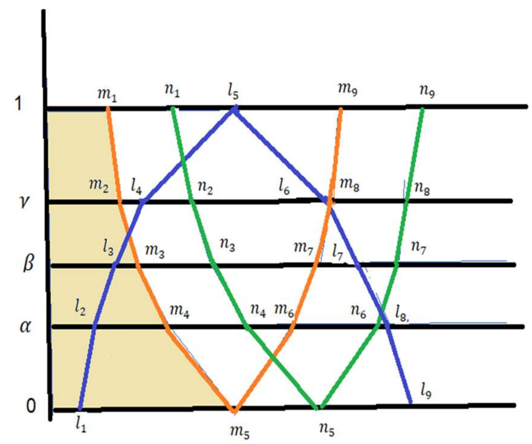


Figure 10

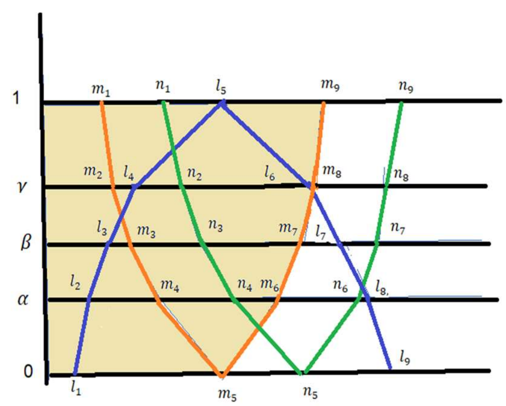


Figure 11

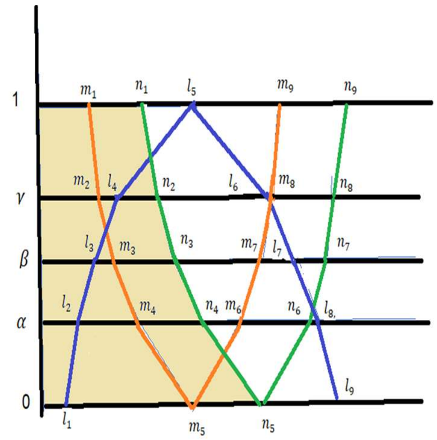


Figure 12

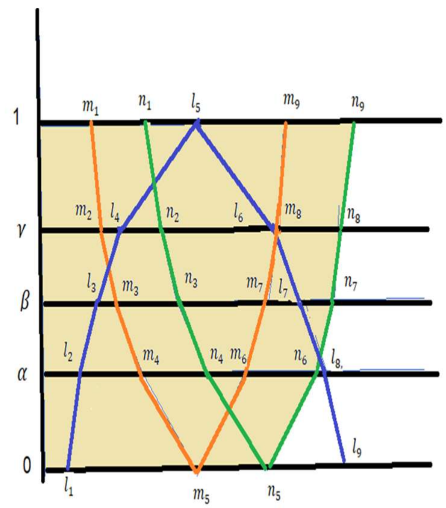


Figure 13

$$A_{neutro}(\breve{X},0) = \text{area of figure 9}$$

$$= \frac{(l_1+l_2)\alpha}{2} + \frac{(l_2+l_3)(\beta-\alpha)}{2} + \frac{(l_3+l_4)(\gamma-\beta)}{2} + \frac{(l_4+l_5)(1-\gamma)}{2}$$

$$A_{neutro}(\breve{X},0) = \text{area of figure 10}$$

$$= \frac{(l_8+l_9)\alpha}{2} + \frac{(l_7+l_8)(\beta-\alpha)}{2} + \frac{(l_6+l_7)(\gamma-\beta)}{2} + \frac{(l_5+l_6)(1-\gamma)}{2}$$

$$A_{neutro}(\breve{Y},0) = \text{area of figure 11}$$

$$= \frac{(m_1+m_2)(1-\gamma)}{2} + \frac{(m_2+m_3)(\gamma-\beta)}{2} + \frac{(m_3+m_4)(\beta-\alpha)}{2} + \frac{(m_4+m_5)\alpha}{2}$$

$$A_{neutro}(\breve{Y},0) = \text{area of figure 12}$$

$$= \frac{(m_8+m_9)(1-\gamma)}{2} + \frac{(m_7+m_8)(\gamma-\beta)}{2} + \frac{(m_6+m_7)(\beta-\alpha)}{2} + \frac{(m_5+m_6)\alpha}{2}$$

$$A_{neutro}(\breve{Z},0) = \text{area of figure 13}$$

$$= \frac{(n_1+n_2)(1-\gamma)}{2} + \frac{(n_2+n_3)(\gamma-\beta)}{2} + \frac{(n_3+n_4)(\beta-\alpha)}{2} + \frac{(n_4+n_5)\alpha}{2}$$

$$A_{neutro}(\breve{Y},0) = \text{area of figure 14}$$

$$= \frac{(n_8+n_9)(1-\gamma)}{2} + \frac{(n_7+n_8)(\gamma-\beta)}{2} + \frac{(n_6+n_7)(\beta-\alpha)}{2} + \frac{(n_5+n_6)\alpha}{2}$$

$$A_{neurto}(D_{hep},0) = \frac{\begin{array}{c}(l_1+l_2+l_8+l_9+m_4+2m_5+m_6+n_4+2n_5+n_6)\alpha + \\ (l_2+l_3+l_7+l_8+m_3+m_4+m_6+m_7+n_3+n_4+n_6+n_7)(\beta-\alpha) + \\ (l_3+l_4+l_6+l_7+m_2+m_3+m_7+m_8+n_2+n_3+n_7+n_8)(\gamma-\beta) \\ (l_4+2l_5+l_6+m_1+m_2+m_8+m_9+n_1+n_2+n_8+n_9)(1-\gamma)\end{array}}{12}$$

Where $0 < \alpha < \beta < \gamma < 1$

When $\alpha = \frac{1}{4}, \beta = \frac{1}{2}, \gamma = \frac{3}{4}$

$A_{neurto}(D_{hep},0) =$

$$= \frac{((l_1+l_9+m_1+m_9+n_1+n_9)+2(l_2+l_3+l_4+l_5+l_7+l_8+m_2+m_3+m_4+m_5+m_6+m_7+m_8+n_2+n_3+n_4+n_5+n_6+n_7+n_8))}{48} \tag{10}$$

4.2.3. Numerical Computation Data for Linear Nonagonal Neutrosophic Numbers

This section presents the numerical output for linear nonagonal neutrosophic numbers, drawing directly from Table 2 in the research material. It says "Heptagonal" instead of the proper "Nonagonal." There are nine points for each of the three membership functions: truth, indeterminacy, and falseness. Each row in this table shows a different nonagonal neutrosophic number, which is defined by its 27 parameters. The table also shows the matching crisp de-neutrosophication value, which is found by using Equation (10). It is important to note a minor typographical error in the original document's table title, which incorrectly states "Heptagonal" instead of the correct "Nonagonal." Each row in this table details a distinct nonagonal neutrosophic number, characterized by its 27 defining parameters (nine points for each of the three membership functions: truth, indeterminacy, and falseness). The table also provides the corresponding crisp de-neutrosophication value, which is derived through the application of Equation (10).

Table 2. Numerical computation of linear Nonagonal neutrosophic number.

S. No	Nonagonal neutrosophic number	De-neutrosophication value
1	(1,1.5,2.5,4,6.5,8.5,10,11,12.5; 2,3,3.5,4.5,7,7.5,8,8.5,9; 5,5.5,6,7,9.5,10.5,11.5,12,13)	7.0729
2	(0.5,1,2,3.5,6,8,9.5,10.5,12; 1.5,2.5,3,4,6.5,7,7.5,8,8.5; 4.5,5,5.5,7,9,10,11,11.5,12.5)	6.5729
3	(2,2.5,3.5,5,7.5,9.5,11,12,13.5; 3,4,4.5,5.5,8,8.5,9,9.5,10; 6,6.5,7,8.5,10.5,11.5,12.5,13,14)	8.0729
4	(0.3,0.8,1.8,3.3,5.8,7.8,9.3,10.3,11.8; 1.3,2.3,2.8,3.8,6.3,6.8,7.3,7.8,8.3; 4.3,4.8,5.3,6.8,8.8,9.8,10.8,11.3,12.3)	6.3729
5	(6.5,7,8,9.5,12,14,15.5,16.5,18; 7.5,8.5,9,10,12.5,13,13.5,14,14.5; 10.5,11,11.5,13,15,16,17,17.5,18.5)	12.572

When we compare the de-neutrosophication values for nonagonal numbers, as shown in Table 2, we see that the behavior patterns are the same as those for heptagonal numbers. The "removal area method" turns this larger complexity into a single, manageable, crisp value, even if there are more parameters (9 points per function for nonagonal vs. 7 for heptagonal). The crisp results usually show the average of the input parameters, which shows that the approach can handle increasingly complex uncertainty representations and is scalable and strong. This stability across various sorts of neutrosophic numbers indicates that the suggested de-neutrosophication method is reliable.

One important thing to note from Table 2 is that the de-neutrosophication values for nonagonal neutrosophic numbers have the same direct relationship with the size of their input parameters as the values for heptagonal numbers. For example, the input parameters for S. No 4 are some of the lowest (e.g., truth function starting at 0.3, indeterminacy at 1.3, falsity at 4.3), while those for S. No 5 are much higher (e.g., truth function starting at 6.5, indeterminacy at 7.5, falsity at 10.5). For S. No. 4, the de-neutrosophication value is 6.37291, while for S.No 5, it is 12.57292. This pattern, where bigger input parameters lead to bigger crisp de-neutrosophication values, is shown in all the samples in Table 2, much like the tendency seen in Table 1 for heptagonal numbers. The fact that neutrosophic numbers (heptagonal and nonagonal) behave in the same way and can be predicted when processed by the same "removal area method" (with different α, β, and γ parameterizations for their geometry) is a strong sign that the method can be used in many situations and is useful in real life. It indicates that the approach dependably turns the "size" or "central tendency" of an uncertain neutrosophic number into a proportionate crisp value. This is important for making decisions and doing meaningful comparative analysis in real life.

5. Results and Discussion

The numerical results shown in Tables 1 and 2 clearly show that the "removal area method" may successfully transform exceedingly complicated linear heptagonal and nonagonal neutrosophic numbers into single, interpretable crisp values. This conversion is crucial because it essentially simplifies the multi-parameter, multi-function representation of uncertainty in these sophisticated neutrosophic numbers to a single, actionable numerical output.

These exact de-neutrosophication values are very helpful in real life. This change makes it feasible for applications that would be extremely hard or impossible to do without it since raw neutrosophic numbers are exceedingly complicated and not like anything else. Some of their most important uses are in multi-criteria decision-making processes, optimization problems, and other areas that need clear and precise numerical inputs. Crisp values make it possible to rank things directly, compare them easily, and easily add them to existing conventional mathematical and computational models. This closes the gap between advanced uncertainty theory and solving real-world problems. This research is a big step forward for the growing discipline of neutrosophic mathematics. This study addresses an important need by giving a clear, proven way and strong numerical examples for de-neutrosophicating new heptagonal and nonagonal neutrosophic numbers. It efficiently connects the theoretical beauty and expressive power of these advanced numbers with their real-world usefulness, which broadens the use and comprehension of neutrosophic theory in many fields of science and engineering. The successful numerical proof of de-neutrosophication for these complex neutrosophic numbers gets rid of a big problem, which opens up new possibilities for creating more advanced algorithms and using them in the actual world. This is a game changer because it lets us create and use new algorithms and models that can fully take use of the better expressive capacity of heptagonal and nonagonal neutrosophic numbers. In industries where uncertainty is a big deal, like financial modeling, medical diagnostics, or complicated engineering systems, this will help people make better decisions, estimate risks more accurately, and optimize more accurately.

6. Conclusion

This research endeavor presents a unique extension of the concept of heptagonal and nonagonal neutrosophic numbers. This example demonstrates the de-neutrosophication technique, which applies the elimination area approach to generate real numbers by converting heptagonal and nonagonal neutrosophic numbers. It will be possible for researchers to develop some intriguing algorithms in the future by using heptagonal and nonagonal neutrosophic numbers. in many different areas, such as difficulties with multi-criteria decision-making and other modeling issues. Once again, under different situations, scientists might develop new structural formulations of heptagonal and nonagonal neutrosophic numbers. In a neutrosophic context, researchers might also compare this study with a freshly developed idea.

The concept of heptagonal and nonagonal neutrosophic numbers is extended in a unique way in this research endeavor. The elimination area technique, a de-neutrosophication process, is used in this work to provide a way to translate heptagonal and nonagonal neutrosophic numbers into real numbers. Future development of intriguing algorithms is made possible by the introduction of heptagonal and nonagonal neutrosophic numbers. A variety of contexts, including multi-criteria decision-making difficulties and other modeling issues, may utilize these figures. The knowledge and possible uses of heptagonal and nonagonal neutrosophic numbers in many situations may be increased by researchers comparing this work with freshly developed concepts in the neutrosophic environment.

To further its application, future studies should concentrate on applying the "removal area method" to additional types of neutrosophic numbers, such as interval or fuzzy neutrosophic numbers. Using a defined benchmark dataset, a thorough comparison of the "removal area method" with other de-neutrosophication methods would offer substantial information about its relative performance and computing efficiency for large-scale challenges. In addition to advancing our theoretical knowledge of neutrosophic mathematics, this strategic plan for future study will greatly increase its applicability in a variety of scientific and technical fields.

Conflict of Interest: The authors declare that they have no conflict of interest.

Re

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