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Diverse Fuzzy Numerical Systems and Their Utilization in the Context of Rankings

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Abstract:

This research article explores the diverse realm of fuzzy numbers, with a specific focus on defuzzification using the bounded area method and its general applications. Fuzzy numbers, introduced by Lotfi A. Zadeh, have proven to be a powerful tool for handling uncertainty in mathematical modelling. The study begins by providing an overview of the foundational concepts of fuzzy numbers, including their types such as Triangular, Trapezoidal, Pentagonal, Hexagonal, and Heptagonal fuzzy numbers. The core of the research delves into the defuzzification process, with a particular emphasis on the bounded area method. This method, introduced for defuzzifying fuzzy numbers, offers a systematic approach to convert fuzzy sets into crisp values by considering the area under the membership function within a defined range. The article explores the mathematical underpinnings of the bounded area method and elucidates its application to various types of fuzzy numbers. Furthermore, the research investigates the general applications of defuzzification by the bounded area method across different domains. From its utilization in control theory to signal processing and decision-making systems, the study highlights the versatility and efficacy of this defuzzification approach. Real-world examples and case studies are presented to underscore the practical significance of employing the bounded area method in diverse applications.

Keywords: Fuzzy Numbers, Pentagonal Fuzzy Numbers, Hexagonal Fuzzy Numbers, Heptagonal Fuzzy Numbers, Octagonal Fuzzy Numbers

2. Fuzzy numbers

2.1. Introduction to Fuzzy numbers:

In the year 1975, Hutton, B (HU) and Rodabaugh, SE (Rod) pioneered the introduction of a revolutionary concept – the fuzzy number. This pivotal notion serves as the cornerstone of the widely employed fuzzy set theory. Specifically chosen from the default fuzzy set encompassing all real numbers, a fuzzy number shares similarities with standard numbers, possessing the attributes of either positivity or negativity within a symmetrically empty space.

Linguistic forms such as 'slightly' or 'quietly' are often employed to articulate the characteristics of fuzzy numbers. The distinctive feature of performing calculations with fuzzy numbers lies in their capacity to incorporate parameters, properties, geometry, and initial conditions within the realm of uncertainty. Within the extensive literature on fuzzy sets, Zadeh (1965) underscores the role of granulation in human cognition, emphasizing how membership functions are meticulously structured to capture individual and subjective human experiences as integral components of a fuzzy set.

A fuzzy set, housing multiple membership functions, operates through a series of well-defined operations within a given universe, denoted by X. These operations are characterized by intervals ranging between 0 and 1, as elegantly illustrated in the following equation. The multifaceted nature of fuzzy numbers, as explored by Hutton and Rodabaugh, has profound implications in various domains, offering a versatile tool for representing and navigating uncertainties inherent in real-world scenarios.

$$\bar{\mu}_{\tilde{A}} : X \rightarrow [0,1]$$

The extent of characterization for an indistinct class involves an infinite set of range values spanning from 0 to 1. The level of characterization for fuzzy numbers, featuring an infinite set of range values within the 0 to 1 spectrum, is precisely defined by the membership function. Fuzzy numbers assume a pivotal role across various domains, including computation, communications products engineering, scientific testing, decision-making, approximate reasoning, and optimization.

2.2 Fuzzy number

A Fuzzy number $\underset{\sim}{A}$ is a fuzzy set on t conditions.

$(i)\ \mu_{\underset{\sim}{A}}(x_0)$ is piecewise continuous.

(ii) There exists atleast one $x_0 \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$.

$(iii)\ \underset{\sim}{A}$ must be normal and convex.

2.3 Triangular fuzzy number

Triangular fuzzy number is defined as $A = (a_1, b_1, c_1)$ where a_1, b_1, c_1 are real numbers and its membership function given below.

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{(x - a_1)}{(b_1 - a_1)} & for\ a_1 \le x \le b_1 \\ \dfrac{(c_1 - x)}{(c_1 - b_1)} & for\ b_1 \le x \le c_1 \\ 0 & for\ x > c_1 \end{cases}$$

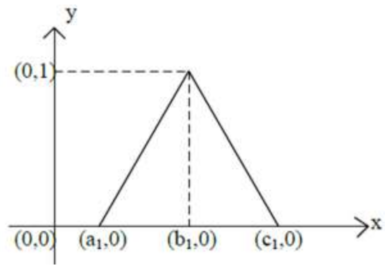

Figure 1: Triangular fuzzy number

2.4 Trapezoidal fuzzy number

Trapezoidal fuzzy number is defined as $A = (a, b, c, d)$ where a, b, c, d are real numbers and its membership function given below.

$$(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{(x - a_1)}{(b_1 - a_1)} & for\ a_1 \le x \le b_1 \\ 1 & for\ b_1 \le x \le c_1 \\ \dfrac{(d_1 - x)}{(d_1 - c_1)} & for\ c_1 \le x \le d_1 \\ 0 & for\ x > d_1 \end{cases}$$

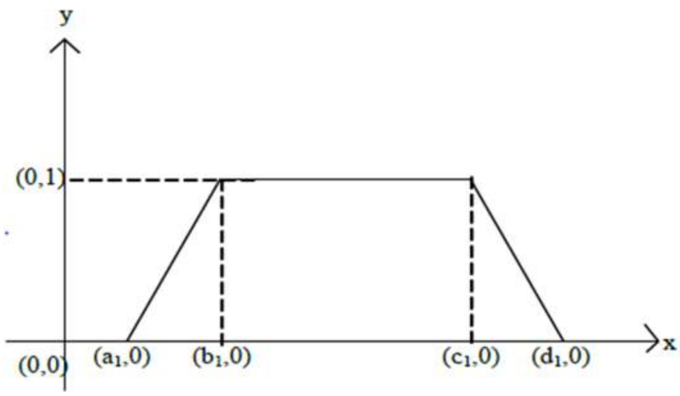

Figure 2: Trapezoidal fuzzy number

2.5 Pentagonal fuzzy number

Pentagonal fuzzy number is defined as $A = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership function given below.

$$\mu_{\tilde{A}_P}(x)=\begin{cases} 0 & ;\ for\ x<a_1, \\ \frac{1}{2}\left(\frac{(x-a_1)}{(a_2-a_1)}\right) & ;\ for\ a_1\le x\le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{(x-a_2)}{(a_3-a_2)}\right) & ;\ for\ a_2\le x\le a_3, \\ 1 & ;\ for\ x=a_3, \\ \frac{1}{2}+\frac{1}{2}\left(\frac{(a_4-x)}{(a_4-a_3)}\right) & ;\ for\ a_3\le x\le a_4, \\ \frac{1}{2}\left(\frac{(a_5-x)}{(a_5-a_4)}\right) & ;\ for\ a_4\le x\le a_5, \\ 0 & ;\ for\ x>a_5. \end{cases}$$

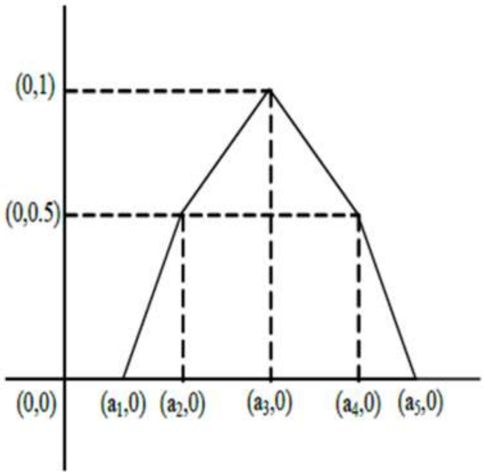

Figure 3: Pentagonal fuzzy number

2.6 Hexagonal fuzzy number

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy numb
a_3, a_4, a_5, a_6 are real numbers and its membershi

$$\mu_{\tilde{A}_H}(x)=\begin{cases} 0 & for\ x<a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1\le x\le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2\le x\le a_3 \\ 1 & for\ a_3\le x\le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for\ a_4\le x\le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5\le x\le a_6 \\ 0 & for\ x>a_6 \end{cases}$$

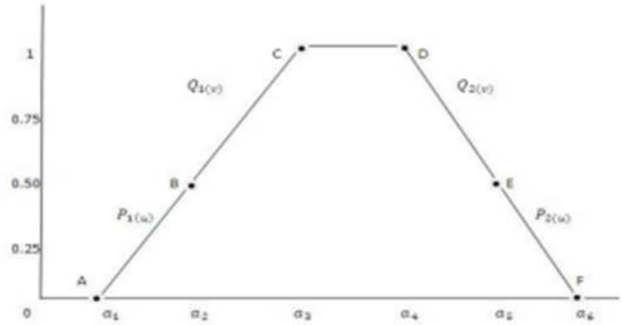

Figure 4: Hexagonal fuzzy number

2.7 Heptagonal fuzzy number

A fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ is a Heptagonal fuzzy number ,if its membership function defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)}, & for\ \ a_1\le x\le a_2 \\ 0.5, & for\ \ a_2\le x\le a_3 \\ \frac{1}{2}+\frac{1}{2}\frac{(x-a_3)}{(a_4-a_3)}, & for\ \ a_3\le x\le a_4 \\ \frac{1}{2}+\frac{1}{2}\frac{(a_5-x)}{(a_5-a_4)}, & for\ a_4\le x\le a_5 \\ 0.5, & for\ \ a_5\le x\le a_6 \\ \frac{1}{2}\frac{(a_7-x)}{(a_7-a_6)}, & for\ \ a_6\le x\le a_7 \\ 0, & otherwise \end{cases}$$

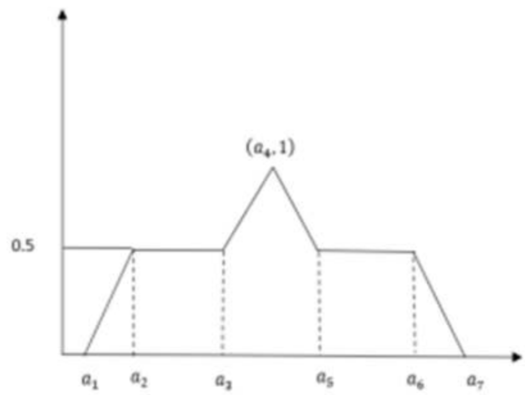

Figure 5: Octagonal fuzzy number

2.8 Defuzzification method Based on the Approach of Bounded Area Method

Defuzzification formula for Trapezoidal fuzzy number

Let us consider a Trapezoidal fuzzy number $T_A = (g_1, g_2, g_3, g_{4;})$

Defuzzification formula can be derived as

$$D_T = \frac{(g_3 + g_4 - g_1 - g_2)}{2}$$

Example : Let us consider a Trapezoidal fuzzy number $T_A = (4,5,8,11)$

Defuzzified value = 5

Let us consider a Trapezoidal fuzzy number $T_A = (g_1, g_2, g_3, g_{4;})$

Customer review	Trapezoidal Fuzzified number	Defuzzified value
5	(6,8,12,13)	5.5
4	(4,5,8,11)	5
3	(3,5,7,10))	4.5
2	(2,5,6,9)	4
1	(1,2,4,6)	3.5

Table 1 : Fuzzified and defuzzified values table for Trapezoidal fuzzy number

Defuzzification formula for Heptagonal fuzzy number:

Let us consider a Heptagonal fuzzy number $T_H = (g_1, g_2, g_3, g_4, g_5, g_6)$

Defuzzification formula can be derived as

$$D_H = \frac{(g_4 + g_5 + g_6 - g_1 - g_2 - g_3)}{3}$$

Example : Let us consider a Heptagonal fuzzy number $T_H = (2,5,6,7,9,10)$

Defuzzified value = 4.33

Customer review	Heptagonal Fuzzified number	Defuzzified value
5	(6,8,12,13,14,17)	6
4	(4,5,8,10,11,12)	5.333333
3	(3,5,7,9,10,10)	4.666667
2	(2,5,6,7,9,10)	4.333333
1	(1,2,4,5,6,7)	3.666667

Table 2: Fuzzified and defuzzified table for Heptagonal fuzzy number

Defuzzification formula for Octagonal fuzzy number:

Let us consider a Octagonal fuzzy number $T_O = (g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8)$

Defuzzification formula can be derived as

$$D_O = \frac{(g_5 + g_6 + g_7 + g_8 - g_1 - g_2 - g_3 - g_4)}{4}$$

Example : Let us consider an Octagonal fuzzy number
T_O =(3,5,7,9,10,10,11,15)

Defuzzified value = 5.5

Table 3 : Fuzzified and defuzzified table for Octagonal fuzzy number

Customer review	Octagonal Fuzzified number	Defuzzified value
5	(6,8,12,13,14,17,19,22)	8.25
4	(4,5,8,10,11,12,15,18)	7.25
3	(3,5,7,9,10,10,11,15)	5.5
2	(2,5,6,7,9,10,13,14)	6.5
1	(1,2,4,5,6,7,8,10)	4.75

For instance, a customer rating of 5 corresponds to the fuzzified number range (6, 8, 12, 13, 14, 17, 19, 22), resulting in a defuzzified value of 8.25. Similarly, ratings of 4, 3, 2, and 1 are associated with their respective fuzzified number ranges, yielding defuzzified values of 7.25, 5.5, 6.5, and 4.75.

The article provides a nuanced understanding of how an extended fuzzified number framework enhances the representation of imprecise customer feedback. By employing advanced defuzzification techniques, the research contributes to refining methodologies for extracting valuable insights from fuzzy data in customer reviews. These findings offer actionable information to businesses seeking to elevate customer satisfaction through data-informed decision-making processes.

The Trapezoidal Fuzzified Number framework is employed to associate customer ratings (ranging from 1 to 5) with specific trapezoidal fuzzy number ranges. The resultant defuzzified values for ratings 1 to 5 are 3.5, 4, 4.5, 5, and 5.5, respectively. This framework provides a baseline for understanding the defuzzification process, emphasizing the representation of customer sentiments within a trapezoidal numerical context.

Moving to the Heptagonal Fuzzified Number framework, the analysis continues with an exploration of heptagonal fuzzy number ranges associated with customer ratings. The derived defuzzified values for ratings 1 to 5 are 3.666667, 4.333333, 4.666667, 5.333333, and 6, respectively. This framework introduces an additional layer of complexity with seven parameters, allowing for a more refined representation of imprecise customer sentiments.

Extending the analysis further, the Octagonal Fuzzified Number framework offers an expanded range for each customer rating. The defuzzified values for ratings 1 to 5 are 4.75, 6.5, 5.5, 7.25, and 8.25, respectively. This framework enhances the flexibility of representation, providing a broader spectrum for interpreting customer feedback and deriving more nuanced defuzzified values.

Through a meticulous examination of these fuzzified number frameworks, the article aims to contribute to the understanding of their distinctive characteristics and practical implications. The findings offer valuable insights for businesses seeking data-informed decision-making processes to enhance customer satisfaction. The comparative analysis provides a foundation for future research and applications in the realm of fuzzy logic, decision sciences, and customer experience analytics.

3. APPLICATIONS OF FUZZY NUMBERS

Triangular fuzzy numbers, a subset of fuzzy numbers, have found widespread applications in diverse fields due to their ability to model and handle uncertainties with simplicity and effectiveness.

In the decision-making, triangular fuzzy numbers prove valuable for representing imprecise information. Examples from fields such as finance, economics, and project management demonstrate their utility in handling uncertain data and facilitating informed decision processes. The article delves into specific methodologies and techniques that leverage triangular fuzzy numbers to enhance decision analysis and optimization.

The engineering domain benefits from the versatility of triangular fuzzy numbers, particularly in systems modelling and control. Illustrative examples showcase how these numbers contribute to robustness in engineering designs, aiding in the development of systems that can adapt to uncertain conditions.

Additionally, the article explores the role of triangular fuzzy numbers in risk assessment and management. Real-world examples highlight their effectiveness in quantifying and mitigating risks across various industries, providing a practical framework for risk analysts.

Trapezoidal fuzzy numbers, a subset of fuzzy numbers, have garnered considerable attention in various domains owing to their efficacy in modelling and managing uncertainty.

The exploration begins with a comprehensive overview of trapezoidal fuzzy numbers, elucidating their fundamental characteristics and representation. Emphasis is placed on the versatility afforded by the four parameters defining their shape, distinguishing them as a valuable tool for expressing imprecise information.

Decision-making processes stand to benefit significantly from the application of trapezoidal fuzzy numbers. Examples drawn from fields such as finance, logistics, and strategic planning illustrate how these fuzzy numbers can adeptly handle imprecise data, facilitating more nuanced and informed decision analyses

Pentagonal fuzzy numbers, a distinctive subset within the realm of fuzzy numbers, have emerged as powerful tools for addressing uncertainty in a wide array of applications. This article aims to explore and elucidate the diverse applications of pentagonal fuzzy numbers, providing insights into their practical utility through illustrative examples.

The exploration commences with a foundational understanding of pentagonal fuzzy numbers, highlighting their unique characteristics and representation. Emphasis is placed on the five parameters defining their shape, distinguishing them as versatile entities capable of expressing complex and nuanced imprecise information.

One prominent domain where pentagonal fuzzy numbers find applicability is in decision-making processes. Illustrative examples drawn from finance, supply chain management, and strategic planning showcase how these fuzzy numbers contribute to more informed decision analyses by adeptly handling imprecision and uncertainty. The article delves into specific methodologies and algorithms that leverage pentagonal fuzzy numbers, demonstrating their potential to enhance decision optimization and risk management.

In engineering and systems modelling, pentagonal fuzzy numbers offer valuable insights into handling uncertainty. Real-world examples highlight their role in designing adaptive systems capable of navigating dynamic and unpredictable conditions, thereby contributing to the robustness and reliability of engineered solutions.

Hexagonal fuzzy numbers, a distinctive subset within the realm of fuzzy numbers, have emerged as versatile tools for modelling uncertainty and imprecision in various applications. This article delves into the extensive applications of hexagonal fuzzy numbers, offering insights into their practical utility through illustrative examples.

The exploration begins with a foundational understanding of hexagonal fuzzy numbers, emphasizing their unique characteristics and representation. The hexagon, defined by six parameters, provides a flexible framework for expressing intricate imprecise information, making hexagonal fuzzy numbers particularly suitable for handling complex uncertainties.

One prominent application domain for hexagonal fuzzy numbers lies in decision-making processes. Examples from finance, project management, and strategic planning demonstrate their effectiveness in facilitating more nuanced decision analyses by adeptly handling imprecision and uncertainty. The article explores specific methodologies and algorithms that leverage hexagonal fuzzy numbers, showcasing their potential to enhance decision optimization and risk management.

In engineering and systems modelling, hexagonal fuzzy numbers contribute to addressing uncertainty in a dynamic environment. Real-world examples highlight their role in designing adaptive systems capable of navigating unpredictable conditions, thereby enhancing the robustness and reliability of engineered solutions.

Hexagonal fuzzy numbers also find applications in forecasting and predictive modelling. Case studies showcase how these fuzzy numbers improve the accuracy of predictions by effectively capturing and representing uncertainties in diverse datasets.

Heptagonal fuzzy numbers, a unique subset within the broader spectrum of fuzzy numbers, offer a distinctive approach to modelling uncertainty and imprecision. This article embarks on an in-depth exploration of the applications of heptagonal fuzzy numbers, shedding light on their versatility through illustrative examples.

The exploration commences with a foundational understanding of heptagonal fuzzy numbers, emphasizing the distinctive features and representation encapsulated by the seven parameters. The heptagon serves as a flexible framework, allowing for the expression of complex imprecise information and making heptagonal fuzzy numbers well-suited for handling intricate uncertainties.

In the realm of decision-making, heptagonal fuzzy numbers prove to be valuable assets. Examples drawn from finance, logistics, and strategic planning illustrate their effectiveness in facilitating nuanced decision analyses by adeptly managing imprecision and uncertainty. The article delves into specific methodologies and algorithms that leverage heptagonal fuzzy numbers, demonstrating their potential to enhance decision optimization and risk management.

Octagonal fuzzy numbers, a unique subset within the broader framework of fuzzy numbers, offer a distinctive means of modelling uncertainty and imprecision. This article undertakes a thorough examination of the applications of octagonal fuzzy numbers, providing insights into their versatile utility through illustrative examples.

The exploration begins with a foundational understanding of octagonal fuzzy numbers, highlighting their distinctive features and representation characterized by the eight parameters. The octagon, as the defining geometric shape, provides a flexible framework for expressing complex imprecise information, making octagonal fuzzy numbers well-suited for handling intricate uncertainties.

In the context of decision-making processes, octagonal fuzzy numbers prove to be valuable tools. Examples drawn from finance, supply chain management, and strategic planning illustrate their effectiveness in facilitating nuanced decision analyses by adeptly managing imprecision and uncertainty. The article delves into specific methodologies and algorithms that leverage octagonal fuzzy numbers, showcasing their potential to enhance decision optimization and risk management.

Applications in engineering and systems modelling showcase the role of octagonal fuzzy numbers in addressing uncertainty within dynamic environments. Real-world examples highlight their contribution to designing adaptive systems capable of navigating unpredictable conditions, thus enhancing the reliability and robustness of engineered solutions.

4. Conclusion:

This research article has systematically explored various fuzzy numbers and their defuzzification through the bounded area method, shedding light on both theoretical foundations and practical applications. The defuzzification process, particularly using the bounded area method, proves to be a reliable technique for converting fuzzy sets into crisp values, enhancing the interpretability of fuzzy logic in decision-making systems. The study emphasizes the significance of understanding and implementing the bounded area method in handling fuzzy numbers of different types, such as Triangular, Trapezoidal, Pentagonal, Hexagonal, and Heptagonal fuzzy numbers. By providing a comprehensive overview of the defuzzification process, this research aims to contribute to the advancement of methodologies for managing uncertainty in mathematical modelling. Moreover, the general applications of the bounded area method showcased in this article underscore its versatility and effectiveness in diverse domains. From control theory to signal processing, the applicability of this defuzzification approach extends to various real-world scenarios. The inclusion of practical examples and case studies reinforces the practical implications of adopting the bounded area method in decision-making processes. This research sets the stage for further exploration and refinement of defuzzification techniques, offering researchers and practitioners a valuable resource for navigating the complexities of fuzzy logic. As the field continues to evolve, the insights presented in this article pave the way for future developments and applications of defuzzification methods in addressing uncertainty across different disciplines.

References:

[1] D.S. Dinagar and S. Kamalanathan, "Sub interval addition method for ranking of fuzzy numbers,"(International Journal of Pure and Applied Mathematics,2017), 115(9),pp. 159–169.

[2] D.S. Dinagar and S. Kamalanathan, "Solving fuzzy linear programming problem using new ranking procedures of fuzzy numbers," (International Journal of Applications of Fuzzy Sets and Artificial Intelligence,2017), 7, pp. 281–292.

[3] S. Dinagar, R.Kamalanathan and N. Rameshan, "Sub Interval Average Method for Ranking of Linear Fuzzy Numbers," (International Journal of Pure and Applied Mathematics, 2017), 114(6), pp. 119–130.

[4] D.S. Dinagar and M.M. Jeyavuthin, "Distinct Methods for Solving Fully Fuzzy Linear Programming Problems with Pentagonal Fuzzy Numbers," (Journal of Computer and Mathematical Sciences, 2019), 10(6), pp. 1253–1260.

[5] S. Dhanasekar, G. Kanimozhi and A. Manivannan, "Solving Fuzzy Assignment Problems with Hexagonal Fuzzy Numbers by using Diagonal Optimal Algorithm,"(International Journal of Innovative Technology and Exploring Engineering,2019), 9(1), pp. 54–57.

[6] K.P. Ghadle and P.A. Pathade, "Solving transportation problem with generalized hexagonal and generalized octagonal fuzzy numbers by ranking method," (Global Journal of Pure and Applied Mathematics,2017), 13(9), pp. 6367–6376.

[7] R. Hassanzadeh, I. Mahdavi, N. Mahdavi-Amiri and A. Tajdin, "An α-cut approach for fuzzy product and its use in computing solutions of fully fuzzy linear systems," (International Journal of Mathematics in Operational Research,2018), 12(2), pp.167–189.

[8] R. Jansi and K. Mohana, "A Heptagonal Fuzzy Number in Solving Fuzzy Sequencing Problem,"(International Journal of Innovative Research in Technology,2018), 5(6), pp. 240–243.

[9] B. Kalpana and N. Anusheela, "Analysis of Fm/Fm/I Fuzzy Priority Queues Based On New Approach of Ranking Fuzzy Numbers Using Centroid of Centroids," (International Journal of Pure and Applied Mathematics,2018), 119(7), pp. 457–465.

[10] A.J. Kamble, "Some notes on Pentagonal fuzzy numbers", (Int. J. Fuzzy Math Arch,2017), 13, pp. 113–121.

[11] S.P. Mondal and M. Mandal,"Pentagonal fuzzy number, its properties and application in fuzzy equation," (Future Computing and Informatics Journal,2017), 2(2), pp. 110–117.

[12] N.I. Namarta, N. Thakur andU.C. Gupta, "Ranking of heptagonal fuzzy numbers using

incentre of centroids," (Int. J. Adv. Technol. Eng. Sci., 2017), 5, pp. 248–255.

[13] M. Prabhavathi, A.Elavarasi, M. Leelavathi and K. Ajitha, "Some Operation on Hexagonal Fuzzy Number," (IJRAR-International Journal of Research and Analytical Reviews (IJRAR),2020), 7(1), pp. 469–482.

[14] A. PraveenPrakash and M. GeethaLakshmi, "A Comparative Study-Optimal Path using Trident and Sub-Trident Forms through Fuzzy Aggregation, Ranking and Distance Methods," (International Journal of Pure and Applied Mathematics,2016), 109(10), pp.139–150.

[15] Ponnialagan,Dhanasekaran, JeevarajSelvaraj and LakshmanaGomathiNayagamVelu, "A complete ranking of trapezoidal fuzzy numbers and its applications to multi-criteria decision making," (Neural Computing and Applications 30, no. 11, 2018), pp. 3303–3315.

[16] P. Rajarajeswari,"Ranking of hexagonal fuzzy numbers for solving multi objective fuzzy linear programming problem,"(International Journal of Computer Applications, 2013), 84(8), pp. 14–19.

[17] P. Rajarajeswari and A.S. Sudha,"Ordering generalized hexagonal fuzzy numbers using rank, mode, divergence and spread," (IOSR Journal of Mathematics,2014), 10(3), pp. 15–22.

[18] C. Rajendran andM. Ananthanarayanan, "Fuzzy Criticalpath Method with Hexagonal and Generalised Hexagonal Fuzzy Numbers Using Ranking Method," (International Journal of Applied Engineering Research, 2018), 13(15), pp. 11877–11882.

[19] M.S. Ramya and M.B.J. Presitha, "Solving an Unbalanced Fuzzy Transportation Problem using a Heptagonal Fuzzy Numbers by Robust Ranking Method," (Journal of Analysis and Computation, 2019), XII(I), pp. 1–13.

[20] K. Selvakumari and G. Sowmiya, "Fuzzy network problems using job sequencing technique in hexagonal fuzzy numbers," (International Journal of Advance Engineering and Research Development,2017), 4(9).

[21] P. Selvam, A. Rajkumar and J.S. Easwari, "Ranking of pentagonal fuzzy numbers applying incentre of centroids," (International Journal of Pure and Applied Mathematics,2017), 117(13), 165–174.

[22] K. Selvakumari and S. Santhi, "A Pentagonal Fuzzy Number Solving Fuzzy Sequencing Problem," (International Journal of Mathematics and its Application,2018), 6(2-B), pp. 207–211.

[23] R. Srinivasan, G. Saveetha and T. Nakkeeran, "Comparative study of fuzzy assignment problem with various ranking," (Malaya Journal of Matematik (MJM), 2020), (1), pp. 431–434.

[24] R. Srinivasan, T. Nakkeeran and G. Saveetha, "Evaluation of fuzzy non-preemptive priority queues in intuitionistic pentagonal fuzzy numbers using centroidal approach," (Malaya Journal of Matematik (MJM),2020), (1), pp. 427–430.

[25] A.M. Shapique, "Arithmetic Operations on Heptagonal Fuzzy Numbers," (Asian Research Journal of Mathematics,2017), pp. 1–25.

[26] A.S. Sudha and S. Karunambigai, "Solving a transportation problem using a Heptagonal fuzzy number," (International Journal of Advanced Research in Science, Engineering and Technology, 2017), 4(1).

[27] SyedaNaziyaIkram and A. Rajkumar, "Pentagon Fuzzy Number and Its Application to Find Fuzzy Critical Path,"(International Journal of Pure and Applied Mathematics, 2017), 114(5), pp. 183–185.

[28] G. Uthra, K. Thangavelu and B. Amutha, "An Approach of Solving Fuzzy Assignment Problem using Symmetric Triangular Fuzzy Number," (International Journal of Pure and Applied Mathematics,2017), 113(7), 16–24.

[29] D.U. Wutsq and N. Insani, "Yager's ranking method for solving the trapezoidal fuzzy number linear programming,"(IOP Publishing, In Jou2rnal of Physics: Conference Series, March, 2018), Vol. 983, No. 1, p. 012135.

[30] Lavanya P "Various fuzzy numbers and various ranking approaches " International Journal of Advanced Research in Engineering and Technology (IJARET) Volume 8, Issue 5,

September - October 2017, pp. 73–82, Article ID: IJARET_08_05_009

[31] Pathinathan. T., Ponnivalavan. K. and Mike Dison. E."Different Types of Fuzzy Numbers and Certain Properties ,Journal of Computer and Mathematical Sciences,Vol.6(11),631-651, November 2015.

[32] Adilakshmi Siripurapu, Ravi Shankar Nowpada, " A new ranking in Heptagonal fuzzy number and its application in project scheduling" RT&A, No 2 (68) Volume 17, June 2022

[33] P. Rajarajeswari , A.Sahaya Sudha and R.Karthika " A new operation on Hexagonal fuzzy number "International Journal of Fuzzy Logic Systems (IJFLS) Vol.3, No3, July 2013.