Performance Comparison Analysis of Piecewise Linear companding for OFDM And WHT Precoded OFDM
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Abstract
Orthogonal Frequency Division Multiplexing (OFDM) is amalgamation of modulation and multiplexing, it helps to allow huge data rates for wireless applications with great spectrum efficiency. Besides of advantages, the uncomfortable issue for OFDM is peak to average power ratio (PAPR). Number of methods was proposed to minimize PAPR, but those are minimizing PAPR at the cost of either increasing the BER, or performance degradation of PSD. In this paper “a composite companding transform by using WHT precoding with piecewise linear companding” is proposed to minimize the PAPR without sacrificing the BER and PSD performances. Simulation results display that this proposed method gives the better trade off between the PAPR minimization and BER performance without sacrificing the PSD.

Keywords: OFDM; PAPR; BER; PSD; WHT; Piecewise linear companding (PLC).

1 INTRODUCTION
OFDM helps to allow huge data rates for wireless applications with great spectral efficiency. Besides of advantages the uncomfortable issue for OFDM is large PAPR. The OFDM system loses either efficiency of the amplifier or PSD performance with this large PAPR. Number of methods was proposed to minimize PAPR [1][2]. Partial transmit sequence (PTS) [3], selective mapping [4], tone reservation (TR) and tone injection [5]. All these techniques minimize the PAPR with high computational and implementation complication. µ-law companding found in [6], signals with small amplitude are enlarged and signals with large amplitude are unaltered to raise the mean power. But its constraint is linear operating region of power amplifier must be increased with the high mean power. Exponential companding was investigated in [7]. EC deals with both signals with large and small amplitudes, maintaining same mean power with extremely high complexity. Later companding with trapezoidal distribution [8], non-linear companding [9] gives better performance on PAPR minimization with PSD performance deviation. Two piecewise linear companding is developed in [10] with low complication but not gives the better PAPR minimization. Lastly piecewise linear companding [11] was proposed. This technique effectively minimizes the PAPR than previously existed techniques with less complexity.

In this paper “performance analysis of piecewise linear companding for WHT Precoded OFDM” on PAPR reduction, BER and PSD performances are discussed.

This paper initialized with the introduction in section 1. Section 2 demonstrates OFDM model and PAPR problem. WHT precoding and piecewise linear companding are discussed in section 3 and 4 respectively. Proposed method is presented in Section 5. Simulation results are displayed in section 6. Lastly conclusion is drawn in section 7.

2 FORMULATION OF PAPR PROBLEM
OFDM is accumulation of ‘N’ sub carriers. QAM or QPSK are used to modulate the data symbols. The discrete time OFDM is given by

$$x_{n} = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} X_{k} e^{j2\pi kn/NL}, \quad 0 \leq n \leq NL−1,$$

(1)

‘N’ is sub carrier size, ‘L’ is oversampling factor generally L is 4 taken.

$$X = \left[ X_{0}, X_{1}, ..., X_{N-1}, 0, ..., 0, X_{N}, ..., X_{N-1} \right]^{T}$$

(2)

PAPR is proportional to size of sub carriers ‘N’. When ‘N’ increases OFDM symbols, power, amplitude follows Gaussian, exponential and Rayleigh distribution respectively. Probability distribution function (PDF) is

$$f_{x}(x) = \frac{2x}{\sigma_{x}^{3}} \exp \left( \frac{-x^{2}}{\sigma_{x}^{2}} \right), \quad x \geq 0.$$  

(3)
The cumulative density function (CDF) of $|X_n|$ is

$$F_{X_n}(x) = \Pr\{|X_n| \leq x\} = \int_0^x \frac{2y}{\sigma} e^{-y^2} \, dy = 1 - e^{-x^2/\sigma^2}, \quad x \geq 0 \tag{4}$$

The PAPR is defined as

$$\text{PAPR}_x = \frac{\max_{n\in[0,N-1]} \left|X_n\right|^2}{E\left[\left|X_n\right|^2\right]} \tag{5}$$

The best way to express PAPR is complementary cumulative distribution function (CCDF) plot. This shows the signals which exceeds specified threshold $\gamma_0 > 0$, i.e.

$$\text{CCDF}_{\gamma_0} = \Pr\{\text{PAPR}_x > \gamma_0\} = 1 - (1 - e^{-\gamma_0})^N \tag{6}$$

3 WALSH HADAMARD TRANSFORM

The Walsh Hadamard transform minimizes the PAPR by reducing the auto correlation of the input data sequence. WHT is simple and real because it has no complex multiplications and the amplitudes of the Walsh functions are +1 and -1.

The Hadamard matrix of order 2 is given by

$$H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{7}$$

Hadamard matrix of $2N$ order is constructed by

$$H_{2N} = \frac{1}{\sqrt{2N}} \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \tag{8}$$

WHT precoded OFDM is obtained by

$$x(t) = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} P[X_k e^{j2\pi k t/T}], \quad 0 \leq t \leq NT \tag{9}$$

Here

$$P = \left[ \begin{array}{cccc} p_m & \cdots & p_{(k-1)m} \\ \vdots & \ddots & \vdots \\ p_{(N-1)m} & \cdots & p_{(N-1)(k-1)m} \end{array} \right] \tag{10}$$

4 PIECEWISE LINEAR COMPAANDING

The companding operation is performed on the original OFDM signal with specified threshold amplitude $A_c$. The piecewise linear companding is shown in fig.1, clips the amplitudes which exceeds the $A_c$ for lessening the peak power, the signals which are near to $A_c$ are transformed linearly for enlarging mean power. The piecewise linear transform function is

$$x(n) = C \{x(n)\} = \begin{cases} x(n) & |x(n)| \leq A_c \\ mx(n) + (1-m)A_c & A_c < |x(n)| \leq A_c, \\ \text{sgn}(x(n)) A_c & |x(n)| > A_c \end{cases} \tag{11}$$

Here $\text{sgn}(x)$ is the sign function.

And the inverse companding function is

$$y(n) = C^{-1}\{r(n)\} = \begin{cases} r(n) & |r(n)| \leq A_c \\ \frac{r(n)}{m} + (1-m)A_c & \frac{|r(n)|}{m} \leq A_c, \\ \text{sgn}(r(n)) A_c & |r(n)| > A_c \end{cases} \tag{12}$$

Here $A_c$, $A_c$ and $m$ are the parameters specified by the Piecewise Linear Companding technique [9]. The theoretical PAPR is obtained by computing $A_c$ by $A_c = \sigma_s 10^{\text{PAPR}_{\text{ Theo}}/10}$, other parameters $A_c$ and $m$ is obtained by solving

$$\int_{A_c}^{A_c} (mx + (1-m)A_c)^2 f_{\sigma_s}(x) \, dx + \int_{A_c}^{\infty} A_c^2 f(x) \, dx = \int_{A_c}^{\infty} x^2 f_{\sigma_s}(x) \, dx. \tag{13}$$

5 PROPOSED TECHNIQUE

A composite companding transform by using WHT precoding and piecewise companding is discussed in this
section. Fig. 2 demonstrates the proposed WHT Precoded OFDM with piecewise linear companding.

Step by step procedure of proposed method

1. Perform WHT transform operations on input data \( X \), by
\[
Y = HX
\]
Where \( H \) is the WHT precoding matrix
\[
P = \begin{bmatrix}
p_{0} & \cdots & p_{(L-1)} \\
\vdots & \ddots & \vdots \\
p_{(N-1)} & \cdots & p_{(N-1)(L-1)}
\end{bmatrix}
\]

2. Perform IFFT operations on WHT processed data by using
\[
y_n = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} Y_k e^{j2\pi \frac{kn}{NL}}, \quad 0 \leq n \leq NL - 1
\]

3. PLC transform is applied to WHT precoded OFDM by
\[
s(n) = C\{y(n)\}
\]
\[
s(n) = C\{y(n)\} = \begin{cases} 
y(n) & |y(n)| \leq A_s \\
sgn(y(n))A_s & |y(n)| > A_s
\end{cases}
\]

4. Inverse PLC transform is applied to received WHT Precoded OFDM with PLC signal \( r(n) \), by using. \( \hat{y}(n) = C^{-1}\{y(n)\} \).
\[
\hat{y}(n) = C^{-1}\{r(n)\} = \begin{cases} 
r(n) & |r(n)| \leq A_s \\
(r(n) - (1-m)A_s)/m & (1-m)A_s \leq |r(n)| \leq A_s \\
sgn(r(n))A_s & |r(n)| > A_s
\end{cases}
\]

5. Then FFT transform operations are performed to \( \hat{y}(n) \), by using \( \hat{Y} = \text{FFT}(\hat{y}) \).

6. Finally inverse WHT transform operations are performed to obtain the output data stream to \( \hat{Y} \).
\[
\hat{X} = H^T\hat{Y}.
\]

6 SIMULATION RESULTS

MAT lab simulated results are furnished to verify the performance of proposed scheme. \( N=256 \) and \( L \) is 4 taken as per WiMAX – IEEE 802.16 standards. \( M \)-QAM (4, 16, 64, 256, and 1024) is used to modulate OFDM symbols. Solid state power amplifier (SSPA), describes the input-output non-linear region characteristics when passed the companded signal via HPA. SSPA model is
\[
|z(t)| = \frac{|y(t)|}{(1 + \frac{|y(t)|^{2p}}{A_{sat}^2p})^{\frac{1}{2p}}}
\]
(14)

\( A_{sat} \) is the saturation level, generally knee factor\( (p) = 2 \) is taken.

Fig. 3 and Fig.4 are CCDF plots of PAPR of WHT Precoded OFDM with PLC and existing companding schemes for 4-QAM & 16-QAM modulation schemes. From Fig. 3 the proposed one attains 0.7 dB for \( PAPR_{\text{preset}} = 4 \text{dB} \), 0.2 dB for \( PAPR_{\text{preset}} = 4.5 \text{dB} \) and 0.1 dB for \( PAPR_{\text{preset}} = 5 \text{dB} \) slightly exceeds the PLC with its corresponding \( PAPR_{\text{preset}} \) at CCDF=10^{-3}.
Fig. 3. CCDF plot of WHT-Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig. 4. CCDF plot of WHT-Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 16-QAM modulation.

Fig. 5. AWGN channel BER performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig. 6. BIT error rate (BER) performance of WHT Precoded OFDM with PLC and existing companding schemes transmitting via AWGN channel for 4-QAM and 16-QAM respectively. WHT Precoded OFDM with PLC is given 0.1 dB slightly better performance than PLC for $PAPR_{\text{preset}} = 4$ dB at the BER level of $10^{-3}$. Fig. 6 describes that BER performance of WHT precoded with PLC for 16-QAM have performance floor at high SNR because of the output of the proposed companding function is not continuous. WHT Precoded OFDM with PLC is given 0.1 dB slightly better performance than PLC for $PAPR_{\text{preset}} = 4$ dB at the BER level of $10^{-2}$. 
Fig. 6. AWGN channel BER performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 16-QAM modulation.

Fig. 7, Fig. 8 demonstrate the Bit error rate (BER) performance of WHT Precoded OFDM with PLC and existing companding schemes transmitting via AWGN channel for 4-QAM and 16-QAM respectively with SSPA model is also adequate.

Table 2 illustrates the SUI Terrain types based on the Tree densities.

<table>
<thead>
<tr>
<th>Terrain</th>
<th>Environment Description</th>
<th>SUI Model</th>
</tr>
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<tbody>
<tr>
<td>C</td>
<td>Flat/Light Tree Density</td>
<td>SUI-1, SUI-2</td>
</tr>
<tr>
<td>B</td>
<td>Flat/Moderate Tree Density</td>
<td>SUI-3, SUI-4</td>
</tr>
<tr>
<td>A</td>
<td>Hilly/Moderate to Heavy Tree Density</td>
<td>SUI-5, SUI-6</td>
</tr>
</tbody>
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Fig. 7 AWGN channel BER performance of WHT Precoded OFDM with original OFDM, WHT Precoded
Fig.10. SUI-2 channel BER Performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig.11. SUI-3 channel BER Performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig.12. SUI-4 channel BER Performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig.13. SUI-5 channel BER Performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig.14. SUI-6 channel BER Performance of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC methods for 4-QAM modulation.

Fig.9-14 demonstrates the BER performance of WHT Precoded OFDM with PLC along with existing companding methods over SUI-1 to SUI-6 Channels for 4-QAM. Fig.9-14 shows that WHT Precoded OFDM with PLC BER performance is slightly improved than PLC.

Fig.15 and fig.16 describes the PSD performance of the WHT Precoded OFDM with PLC and PLC along with original OFDM for 4-QAM and 16-QAM. PSD is obtained by Non-parametric estimation.

Fig. 15. PSD plot of WHT Precoded OFDM with original OFDM, WHT Precoded OFDM and PLC techniques for 4-QAM modulation.
7 CONCLUSION
In this paper, a composite companding transform by using WHT Precoded OFDM with Piecewise linear companding transform is proposed to minimize PAPR value of OFDM while maintaining the BER performance over various channels such as AWGN, SUI 1-6 channels. Simulation results verify that the proposed technique marginally reduces the PAPR value compared to the Piecewise linear companding (PLC) technique and slightly improves the BER performance over AWGN and SUI channels except for SUI-1 and SUI-2 channel for both 4-QAM and 16-QAM modulations without sacrificing the PSD performance with excess computational complexity.

REFERENCES